

Theoretical model for predicting thermodynamic behavior of thermal-lag Stirling engine

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ABSTRACT

A theoretical model for predicting thermodynamic behavior of thermal-lag Stirling engine is presented in this study. Without a displacer and its link, the thermal-lag engine contains only a moving part (piston) and a static part (regenerative heater) in engine's cylinder and hence, is regarded as a unique type of Stirling engines that featuring rather simple mechanical structure. In this study, a numerical simulation of thermodynamic behavior of the thermal-lag Stirling engine is performed based on the theoretical model developed. Transient variations of temperatures, pressures, pressure difference, and working fluid masses in the individual working spaces of the engine are predicted. Dependence of indicated power and thermal efficiency on engine speed has been investigated. Then, optimal engine speeds at which the engine may reach its maximum power output and/or maximum thermal efficiency is determined. Furthermore, effects of geometrical and operating parameters, such as heating and cooling temperatures, volumes of the chambers, thermal resistances, stroke of piston, and bore size on indicated power output and thermal efficiency are also evaluated.

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1. Introduction

Stirling engines have been regarded as promising power generation method since they have been successfully applied in a wide range of energy applications, including power plants [1] and hybrid automotive engines [2]. Therefore, in the past several decades, numerous studies [3–6], were presented to investigate the performance of Stirling engines and their optimization as well. Conventionally, Stirling engines are classified into three major types, namely, α -, β - and γ -types, in terms of their configurations which are illustrated in Fig. 1. The α -type engine is equipped with two cylinders and each of the two cylinders contains a piston inside. The β -type engine has only a single cylinder and the cylinder contains one piston and one displacer which are placed coaxially. The γ -type engine is also equipped with two cylinders; however, one cylinder contains a piston and another cylinder contains a displacer. In practices, relative movement between the two moving parts (piston–piston or displacer–piston) inside the cylinders is carefully designed by assigning a phase angle between the two moving parts. In this manner, the working fluid is able to flow back-and-forth between the heating and the cooling parts of the engine to absorb or reject heat, respectively, and then complete

a thermodynamic cycle. Detailed information regarding the configurations of different Stirling engines is available in Ref. [7]. Relative performance of these three major types of the Stirling engines was investigated by Cheng and Yang [8].

Recently, one other configuration of Stirling engine was proposed and was referred to as thermal-lag Stirling engine. The schematic of the thermal-lag Stirling engine is shown in Fig. 1(d). A thermal-lag Stirling engine requires no displacer as well as its connected link, and instead, a porous-medium block is fixed in the cylinder as a static regenerative heater. The regenerative heater may be made of metal wires, screen mesh, or metal foams, which serves not only for heat absorption but also for resulting in a phase angles in the periodic variations of volumes and pressures. Without the displacer and its link, the thermal-lag engine has a moving part (piston) and a static part (regenerative heater) in engine's cylinder only and hence, its mechanical structure can be greatly simplified. However, it is important to mention that the power density and the thermal efficiency of the thermal-lag engines are relatively low compared to other types of Stirling engines because without a displacer the working fluid might not be swept effectively between the heating and the cooling parts. Therefore, how to increase the performance of the thermal-lag engines becomes a major concern to the related researchers.

It is important to mention that the configurations of the thermal-lag and the thermoacoustic engines [9,10] look similar. However, the difference between the thermal-lag and the

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Nomenclature

A_r	cross section void area of regenerative heater (m^2)	T'_h, T'_l	temperature of working fluid exiting regenerative heater (K)
c_p	constant pressure specific heat (J/kg K)	$\bar{u}$	average flow velocity in regenerative heater (m/s)
c_v	constant volume specific heat (J/kg K)	v_p	velocity of piston (m/s)
d	diameter (m)	V	volume (m^3)
D_h	hydraulic diameter of regenerative channel (m)	$V_{\max}/V_{\min}$	compression ratio
e	dimensionless parameter, $= l_{PA}/r$	V_{ps}	piston swept volume (m^3)
f_F	Fanning friction factor	V_{tot}	total volume, $= V_c + V_t + V_r + V_g$ (m^3)
f	direction factor	W_i	indicated power (W)
F_f	friction factor	x	axial coordinate in regenerative heater (m)
h_r	heat transfer coefficient of regenerative heater ($\text{W/m}^2 \text{K}$)	y_p, y_{ps}	heights of piston (m)
k	thermal conductivity of working fluid (W/m K)	Greek symbols	
K_f	dimensionless parameter, $= F_f Re_D$	η_t	thermal efficiency
l	length (m)	μ	dynamic viscosity (N s/m^2)
m	mass of working fluid (kg)	ρ	density of working fluid (kg/m^3)
$\dot{m}$	mass flow rate through regenerative heater (kg/s)	ϕ	crank angle (rad)
p	pressure (Pa)	ω	engine speed (rpm)
p_0	charged pressure (Pa)	ω_o	engine speed (rad/s)
P	perimeter of regenerative heater channel (m)	Subscripts	
$\dot{Q}_{\text{in}}$	total heat input rate (W)	ave	average
R	gas constant, (J/kg K)	c	compression chamber
R	thermal resistance (K/W)	g	gas chamber
Re_D	Reynolds number, $= \rho \bar{u} D_h / \mu_l$	h	higher-temperature space
r	offset distance from the crank to the center of shaft (m)	in	flow in
t	time (s)	out	flow out
$T_{\text{wh}}, T_{\text{wl}}$	wall temperatures of lower- and higher-temperature spaces (K)	l	lower-temperature space
T_h, T_l	working fluid temperatures in lower- and higher-temperature spaces (K)	r	regenerative heater
		t	thermal buffer chamber
		w	wall

thermoacoustic engines could be observed based on their pressure variations. In theory, the pressure in the working space of both the two engines can be approximated by $p_m + p'$, where p_m is the ambient pressure (zero order) and p' is the thermoacoustic pressure (first order). In the thermoacoustic engine, the ambient pressure is always assumed constant since the change of the working space volume is small during operation, but the acoustic pressure is varied due to the temperature gradient built in the stack. The power of the thermoacoustic engine is mainly produced by the acoustic pressure (p') and volume variation of the small gas parcels in the stack or regenerator, and is transferred by the gas parcels in the working space. Thus, in the thermoacoustic engine, the variation of working space is so small that its power is typically transmitted by membranes of the transducers or microphones.

On the contrary, in a thermal-lag engine, the variation in the volume of the working space is much larger due to the movement of piston (the compression ratio may reach over 1.6). In this situation, the ambient pressure (or should be called main pressure in the working space) is no longer constant but varied due to working space volume variation and heating/cooling as well. And, the power of the thermal-lag engine is mainly produced by the ambient pressure (p_m) and volume variation of the working space. It is important to note that in the thermal-lag engine the variation in the ambient pressure reaches an order of several atms, whereas the variation in the acoustic pressure is just within a few kPas. That is, the ambient pressure variation is typically over 100 times the acoustic pressure variation. The acoustic pressure variation caused by the thermoacoustic effects is so small that it definitely can be neglected in the present engine we designed.

Furthermore, in a thermal-lag engine, a piston of sufficient mass is installed and connected to the flywheel through a link in order to

drive the working gas and deliver power as well. With the piston assembly (including piston, link, flywheel, external loads, etc.), the piston and the working gas are oscillating at relatively low frequency (say, 8–15 Hz). On the other hand, in a thermoacoustic engine, the working gas is driven by the thermoacoustic wave generated due to a temperature gradient formed in the porous-medium stack. The piston used in the thermoacoustic engine is typically the membranes of the transducers or microphones such that the membrane is able to oscillate in resonance with the thermoacoustic wave of relatively high frequency (say, 200–450 Hz).

To authors' knowledge, the concept of the thermal-lag Stirling engine was originally proposed by Chen and West [11]. Their conceptual design firstly reveals the fundamental structure of the thermal-lag engines. A hole was made at the cold side of the engine's cylinder for exhausting high temperature working fluid. Lately, the first prototype was successfully built by Taler [12], and later Taler [13] tested the performance of the thermal lag engine and compared it with the equivalent traditional Stirling engines. In Taler's engine, there was no gas port in the cylinder, and the working space was a closed volume. According to the comments of Taler [12,13], the engine experiences imperfect cooling and heating processes in the working spaces, and the imperfect cooling and heating cause a time lag in the gas temperature response as following the movement of the piston. Thus, the engine was named 'thermal-lag' engine thereafter. West [14] presented theoretical solutions for the power output of Taler's engine by performing a simple thermodynamic analysis. The author divided the whole engine into two regions: cylinder and regenerative heater, and calculated indicated works associated with isothermal and adiabatic processes individually. Arques [15] provided a numerical model for the thermal-lag engine by solving energy equation and

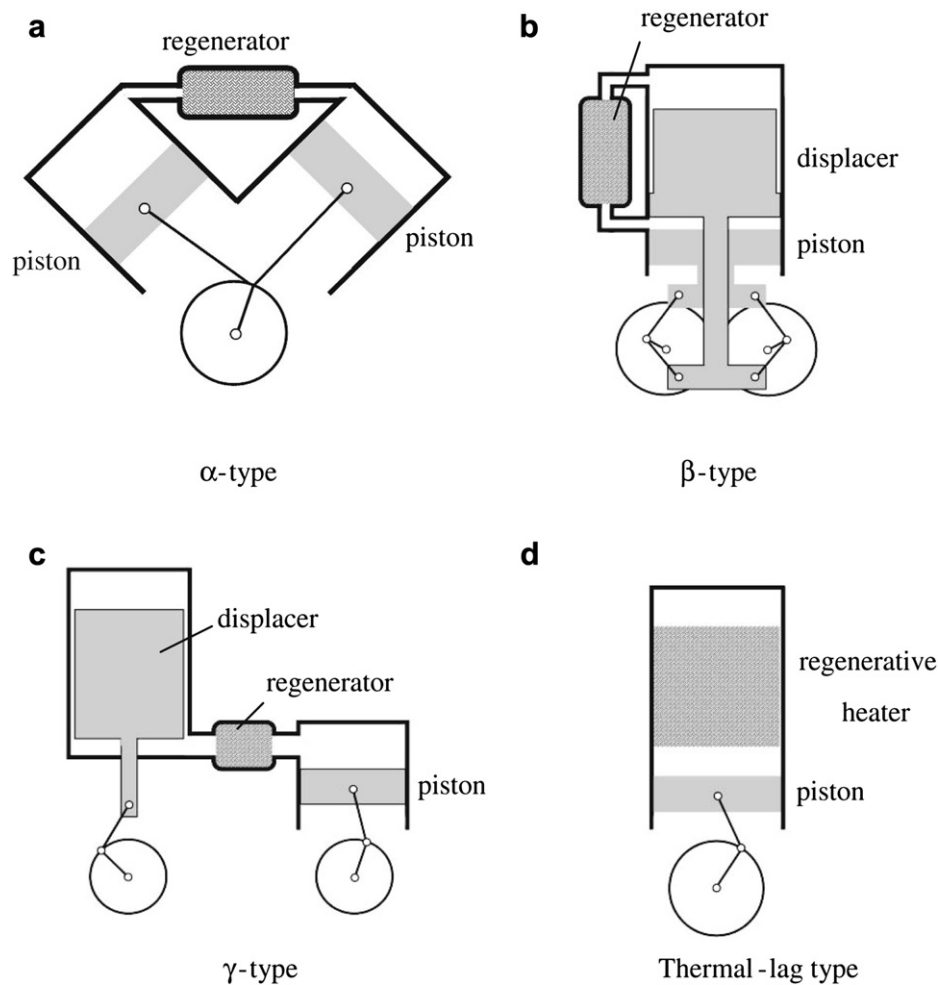


Fig. 1. Configurations of α -, β -, γ -, and thermal-lag-type Stirling engines. (a) α -type (b) β -type (c) γ -type (d) thermal-lag-type.

considering the pressure drop through the regenerative heater. One may find that the areas of the thermodynamic cycles plotted in the p – V diagrams of this report are very small. It seemingly appears that the thermal-lag engines are not capable of producing appreciable work output.

However, recently the present group of authors successfully built a small prototype thermal-lag engine using air as the working fluid. The photograph of the thermal-lag engine and its parts are provided in Fig. 2, and the geometrical and operating parameters are listed in Table 1. Based on what we have learned about the engine, authors firstly observed that the thermal-lag engine is actually capable of producing appreciable power output of over 15 W. As listed in Table 1, for the thermal-lag engine considered in the present study, the rotation speed is 500 rpm (i.e., the piston is oscillating approximately at 8 Hz, much lower than the thermoacoustic frequency), and the stroke of the piston is 0.05 m ($=4V_{ps}/\pi d^2$, much higher than the penetration depth caused by the thermoacoustic wave). In this situation, the effects of the thermoacoustic wave can be negligible.

The power output is further increased as the charged pressure of air is elevated. Secondly, due to the simplicity of the mechanism, design of the dimensions of the engine parts becomes more critical. A smart geometrical design may significantly improve the performance of the engine. When designing the engine, one definitely desires a numerical simulation tool that helps predict the performance of the engine under various geometrical conditions. The

numerical simulation tool stemming from theoretical analysis may produce low-cost, detailed data that are not easily obtained by experiments. However, compared to the studies of traditional Stirling engines [16,17], substantial theoretical investigation of the thermal-lag Stirling engines is severely insufficient. Because of the lack of theoretical study, fundamentals of the theory of the thermal-lag engines are not completely clarified. Under these circumstances, in this study a theoretical model for predicting the thermodynamic behavior of the thermal-lag Stirling engine is presented. Meanwhile, numerical simulation of the performance of the thermal-lag Stirling engine is performed based on the theoretical model developed.

Probably the most appealing advantage of the thermal-lag engine is the low cost and easy maintenance. The thermal-lag engine has only one moving piston and the piston is actually operating in the low-temperature space. Hence, the mechanical structure can be greatly simplified and the reliability of the engine is improved. Besides, it has been observed from the experiments of the prototype engine that the engine can be easily pushed start. When an extremely small rotation speed is applied, the engine initially exhibits a 'bi-directional swinging mode' in a short period of time and then proceeds to a 'uni-directional rotating mode.' It implies that the engine is possible to start at relatively lower electricity consumption. These advantages make this engine competitive particularly when it is applied in a small scale concentrating solar power (CSP) system for converting solar energy



a Prototype engine



b Original parts

Fig. 2. A 15-W thermal-lag Stirling prototype engine with crank-drive mechanism and its original parts. The working fluid is air at 1 atm. (a) Prototype engine (b) original parts.

into electricity in the mountains or other remote areas where regular maintenance is difficult.

2. Theoretical model

The geometrical parameters of a typical thermal-lag Stirling engine are illustrated in Fig. 3. The whole space in the cylinder is divided into four chambers: gas chamber (V_g), regenerative heater (V_r), thermal buffer chamber (V_t), and compression chamber (V_c). As the only moving part (piston) is moving between top-dead and

Table 1

Geometrical and operating parameters of base-line case.

Geometrical parameters	
r (m)	0.025
e	3
d (m)	0.045
A_r (m ²)	1.988×10^{-4}
V_t (m ³)	5×10^{-6}
V_g (m ³)	30×10^{-6}
V_{ps} (m ³)	79.522×10^{-6}
$V_{\max}/V_{\min}$	1.691
Operating parameters	
ω (rpm)	500
T_{wrl} (K)	300
T_{wrh} (K)	1000
T_{wl} (K)	300
T_{wh} (K)	1000
R_h (K/W)	1
R_r (K/W)	200
R_c (K/W)	1
p_0 (Pa)	101,325

bottom-dead points, the volume of the compression chamber is varied with time, while the volumes of gas chamber, regenerative heater chamber and thermal buffer chamber are fixed. Wall temperature of the gas chamber is maintained at T_{wh} due to heating by an external high-temperature reservoir, and on the other hand, that of the thermal buffer chamber and the compression chamber is maintained at T_{wl} due to external cooling. Therefore, herein the gas chamber is called higher-temperature space, and the combined space of the thermal buffer chamber and the compression chamber is called lower-temperature space. An axial temperature gradient is hence established within the regenerative heater.

As illustrated in Fig. 3, the distance between upper surface of piston and the center of shaft y_{ps}^k can be written as:

$$y_{ps}^k = l_p + r \sin \phi^k + r \sqrt{e^2 - \cos^2 \phi^k} \quad (1)$$

where ϕ is the crank angle and e is l_{pA}/r . Differentiate Eq. (1) with respect to t yields the velocity of piston v_p^k as:

$$v_p^k = r \omega_o \cos \phi^k \left(1 + \sin \phi^k / \sqrt{e^2 - \cos^2 \phi^k} \right) \quad (2)$$

The piston swept volume (V_{ps}) is determined with $V_{ps} = \pi r d^2 / 2$, and the volumes of thermal buffer chamber, gas chamber and regenerative heater can be calculated by $V_t = \pi l_t d^2 / 4$, $V_g = \pi l_g d^2 / 4$, $V_r = A_r l_r$, where A_r is the cross section void area of the regenerative heater. The volume of the lower-temperature space (V_l) is

$$V_l^k = V_t + V_c = V_t + \frac{V_{ps}}{2} \left(1 + e - \sin \phi^k - \sqrt{e^2 - \cos^2 \phi^k} \right) \quad (3)$$

Heat transfer rates in the lower- and higher-temperature spaces can be calculated provided that temperature differences between the working fluids and the walls of the chambers, such as $T_{wh} - T_h$ and $T_l - T_{wl}$, and the thermal resistances are obtained. The thermal resistance in the lower-temperature space is expressed by

$$R_l^k = \left[\frac{1}{R_t} + \frac{1}{R_c} \left(1 + e - \sin \phi^k - \sqrt{e^2 - \cos^2 \phi^k} \right) \right]^{-1} \quad (4)$$

where R_t and R_c denote the thermal resistances in the thermal buffer chamber and the compression chamber. On the other hand, the thermal resistance in the higher-temperature space is denoted

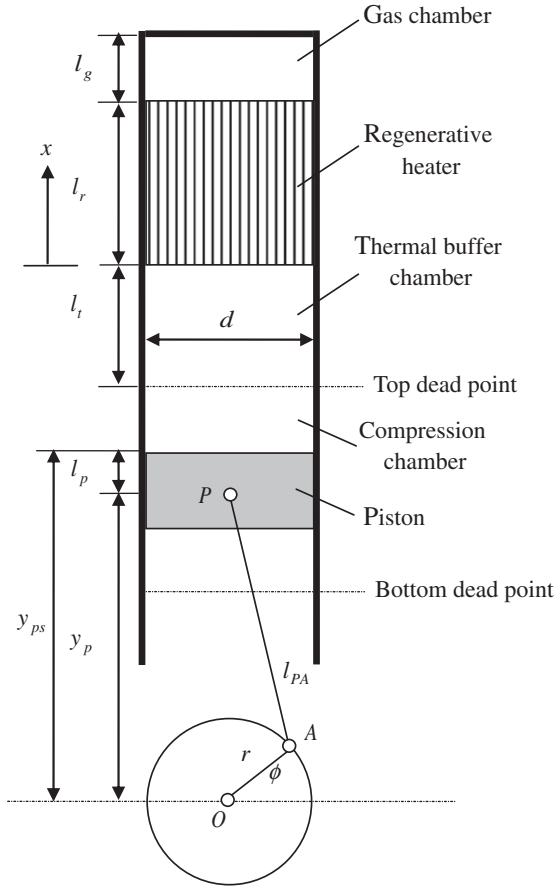


Fig. 3. Schematic of prototype engine.

by R_h . Typically, R_t , R_c and R_h are tuned according to experimental data. In this study, they are regarded influential parameters and their effects on the performance of the engine are evaluated. The values of R_t , R_c and R_h for the base-line case are listed in Table 1.

Mass flow rate through the regenerative heater is dependent on the pressure difference between the higher- and lower-temperature spaces and the properties of the porous medium as well. For a regenerative heater which is composed of metal form or wire-mesh woven-screen matrices, the pressure drop can be expressed as a quadratic function of average velocity or expressed in dimensionless form as a Fanning friction factor F_f [18–22]. The Fanning friction factor is defined as

$$F_f = \frac{D_h}{l_r} \frac{\Delta p}{\rho \bar{u}^2} \quad (5)$$

where $\Delta p = p_l^k - p_h^k$ and hydraulic diameter $D_h = 4A_r/P$.

The form of the relation between the Fanning friction factor and the Reynolds number corresponding to fully developed laminar flows in microchannels [23] is adopted here as:

$$F_f Re_D = K_f \quad (6)$$

where K_f is an empirical coefficient. The relation between average velocity and pressure drop can then be derived from Eqs. (5) and (6) as

$$\bar{u}^k = -\xi_u (p_h^k - p_l^k) \quad (7)$$

where $\xi_u = D_h^2 / 2\mu K_f l_r$. The mass flow rate through the regenerative heater can be obtained as:

$$\dot{m}^k = \rho_{tg}^k A_r \bar{u}^k \quad (8)$$

where ρ_{tg}^k is density of working fluid within the regenerative heater which is determined with upstream conditions. The average velocity is assigned to be positive when the working fluid flows from the thermal buffer chamber to the gas chamber. Therefore, if $\bar{u}^k > 0$, $\rho_{tg}^k = \rho_l^k$, and if $\bar{u}^k < 0$, $\rho_{tg}^k = \rho_h^k$. The masses of the working fluid contained in the lower- and the higher-temperature spaces at each time step can be calculated by

$$\begin{cases} m_l^{k+1} = m_l^k - \dot{m}^k \Delta t \\ m_h^{k+1} = m_h^k + \dot{m}^k \Delta t \end{cases} \quad 9a \text{ and } b$$

where Δt denotes the time step.

Next, heat transfer rate to or from the working fluid while it is flowing through the regenerative heater must be evaluated. Traditionally, the heat transfer performance of a regenerator is measured in terms of effectiveness of the regeneration [24–26]. However, the effectiveness of a regenerator is really involved. The magnitude of it is dependent on the geometry, properties of the porous material, engine speed, and temperature conditions at both ends of the regenerator [5,27]. In the present study, the concept of the effectiveness of regenerator is not adopted. Instead, a direct derivation for the axial temperature profile of the regenerative porous matrix is attempted. Axial temperature profiles within the regenerative heater are displayed in Fig. 4. The axial temperature profile of the regenerative porous matrix is assumed to be a steady and linear function varied from T_{wrh} (hot side) to T_{wrl} (cold side). When the working fluid passes through the regenerative heater, the enthalpy change of the working fluid is mainly caused by the convective heat transfer to or from the working fluid. Therefore,

$$\rho_{tg}^k c_p \bar{u}^k A_r dT = h_r P dx (T_{wr} - T) \quad (10)$$

where T_{wr} is the temperature of the regenerative porous matrix, and P is the cross section void perimeter of the regenerative porous matrix. Since the axial temperature profile of the regenerative porous matrix is assumed to be a steady and linear function varied

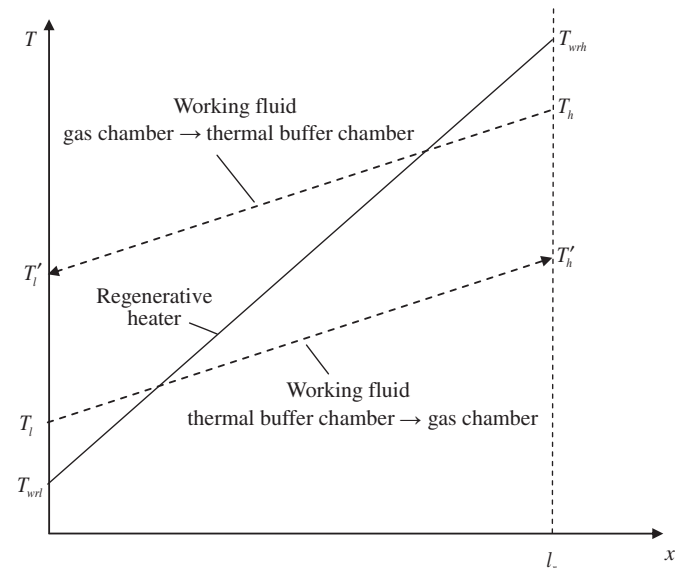


Fig. 4. Axial temperature profiles within regenerative heater.

from T_{wrh} to T_{wrl} , one has $T_{wr} = -T_{wrl} - T_{wrh}/l_r x + T_{wrl}$. From Eq. (10), one obtains

$$\frac{dT}{dx} = a^k (T_{wr} - T) \quad (11)$$

where $a^k = 4h_r/\rho_{tg}^k c_p \bar{u}^k D_h$. Note that the heat transfer coefficient h_r can be evaluated by using the information of Nusselt number provided in Refs. [23,28].

As the working gas flows from the lower- to the higher-temperature space, the working gas absorbs heat from the regenerative heater and hence its temperature rises from T_l^k to T_h^k . On the contrary, while the working gas flows from the higher- to the lower-temperature space, it is cooled and its temperature falls from T_h^k to T_l^k . Therefore, the boundary conditions associated with Eq. (11) is described as:

(1) If $\bar{u}^k > 0$, $T = T_l^k$ at $x = 0$:

One obtains the temperature distribution of the working fluid in the regenerative heater as:

$$T(x) = (T_l^k - T_{wrl}) e^{-a^k x} - \frac{T_{wrh} - T_{wrl}}{l_r} \left(\frac{1 - e^{-a^k x}}{a^k} - x \right) + T_{wrl} \quad (12)$$

Let $x = l_r$, and then Eq. (12) gives the temperature of working fluid at the exit of the regenerative heater to the hot side

$$T_l^{k+1} = T_{wrh} - \frac{1 - e^{-a^k l_r}}{a^k l_r} (T_{wrh} - T_{wrl}) + (T_l^k - T_{wrl}) e^{-a^k l_r} \quad (13)$$

(2) If $\bar{u}^k < 0$, $T = T_h^k$ at $x = 0$:

Similarly, one may obtain the temperature of working fluid at the exit of the regenerative heater to the cold side

$$T_l^{k+1} = T_{wrl} + \frac{1 - e^{-a^k l_r}}{a^k l_r} (T_{wrh} - T_{wrl}) - (T_{wrh} - T_h^k) e^{-a^k l_r} \quad (14)$$

On the other hand, transient temperature variation of the working fluid in the lower-temperature space can be predicted based on the energy conservation law. In finite-difference form, it is expressed as

$$T_l^{k+1} = \frac{m_l^k}{m_l^{k+1}} T_l^k - \frac{p_l^k}{m_l^{k+1} c_v} (V_l^{k+1} - V_l^k) + \frac{\Delta t}{m_l^{k+1} c_v} \times \left[\frac{T_{wl} - T_l^k}{R_l^k} - \dot{m}^k c_p (f_{tg}^k T_l^k + f_{gt}^k T_l^k) \right] \quad (15)$$

On the right-hand side of the above equation, the second term is the work done by the working fluid, and the first term in bracket denotes the heat transfer which is calculated in terms of thermal resistance and temperature difference. In addition, the second term in the bracket represents energy transfer rate accompanying the mass flow, and hence, it is dependent on the direction of the mass flow. The two direction factors f_{tg}^k and f_{gt}^k appearing in Eq. (15) are introduced to show the direction effect, which are defined as

$$f_{tg}^k = \begin{cases} 1 & \text{if } \bar{u}^k > 0 \\ 0 & \text{if } \bar{u}^k \leq 0 \end{cases}, \quad f_{gt}^k = \begin{cases} 1 & \text{if } \bar{u}^k < 0 \\ 0 & \text{if } \bar{u}^k \geq 0 \end{cases} \quad (16)$$

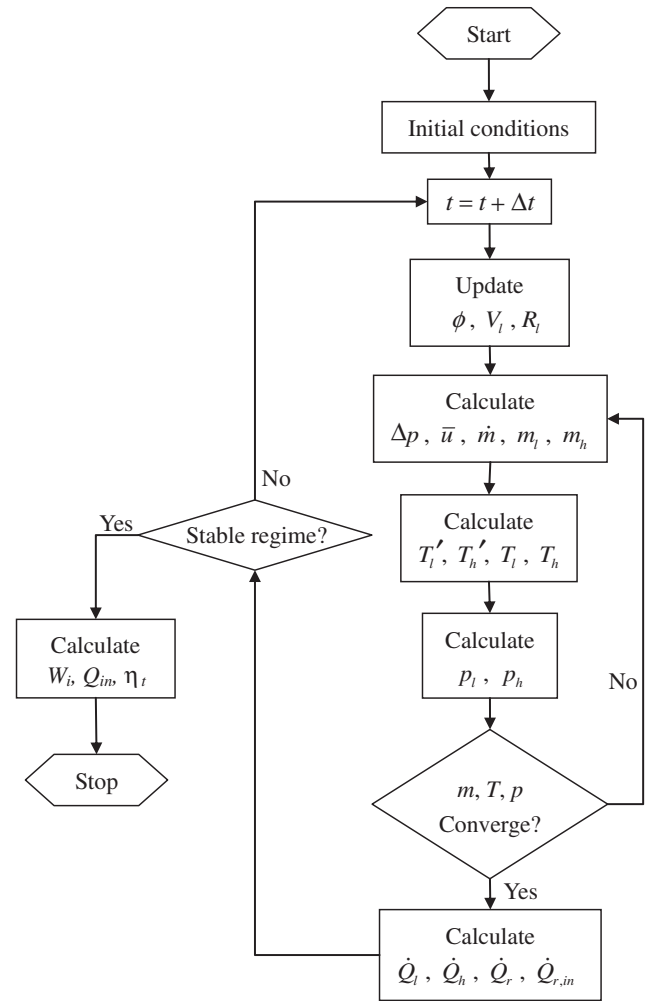


Fig. 5. Flow chart of computation process.

Similarly, temperature of the working fluid in the higher-temperature space can be calculated by:

$$T_h^{k+1} = \frac{m_h^k}{m_h^{k+1}} T_h^k + \frac{\Delta t}{m_h^{k+1} c_v} \left[\frac{T_{wh} - T_h^k}{R_h} + \dot{m}^k c_p (f_{tg}^k T_h^k + f_{gt}^k T_h^k) \right] \quad (17)$$

There is no moving-boundary work done by the working fluid in the higher-temperature space because the volume of the gas chamber is fixed. With the ideal-gas equation of state, the pressures in the lower- and the higher-temperature spaces are calculated, respectively, as

$$p_l^{k+1} = \frac{m_l^{k+1} R T_l^{k+1}}{V_l^{k+1}}, \quad p_h^{k+1} = \frac{m_h^{k+1} R T_h^{k+1}}{V_h} \quad (18)$$

Note that here $V_h = V_g$.

Indicated work per cycle is determined by integrating the stable loop of the cycle in p - V diagram based on the p - V solutions in the lower-temperature space. Then, the indicated power of the engine is determined by multiplying the indicated work per cycle and the engine speed as

$$\dot{W}_i = \frac{\omega_0}{2\pi} \oint p_l^k dV_{tp} \quad (19)$$

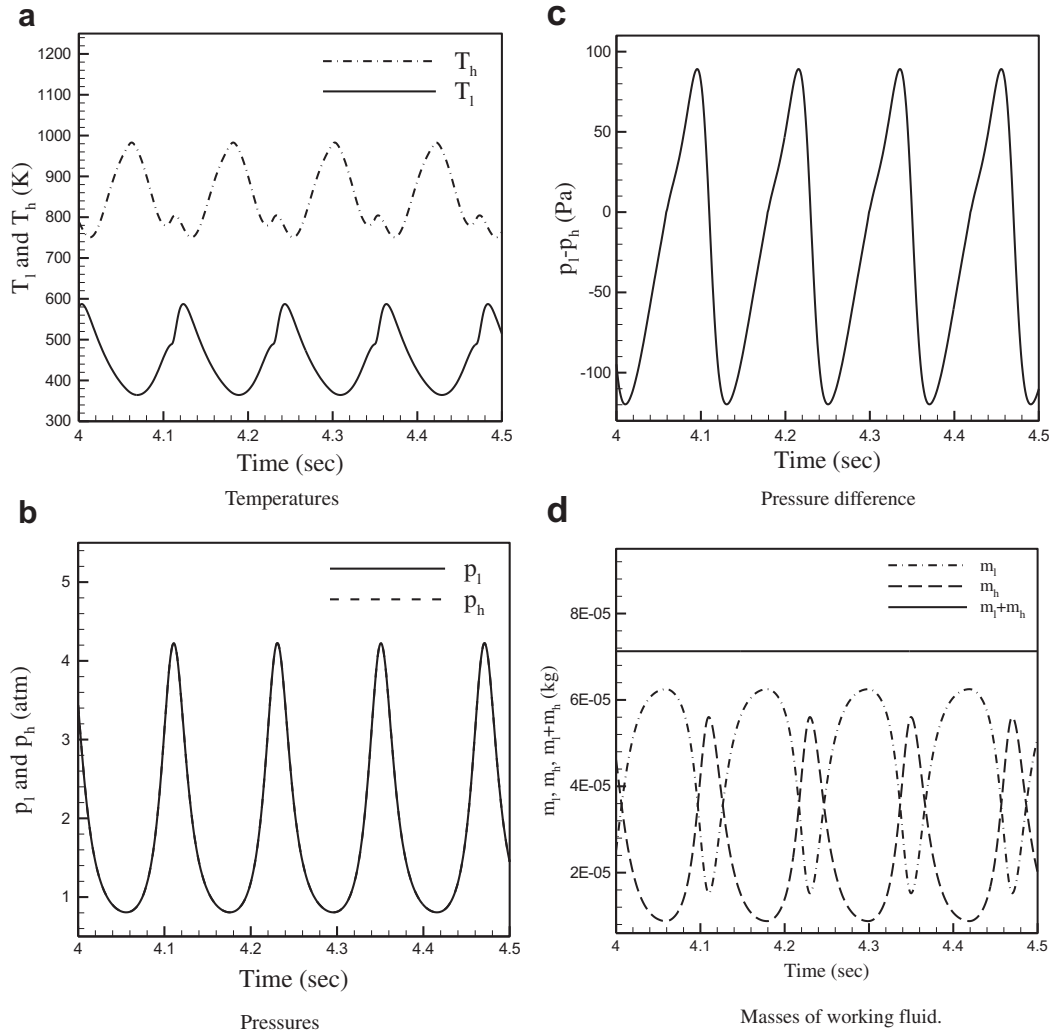


Fig. 6. Stable variations of thermodynamic properties in lower- and higher-temperature spaces, for base-line case. (a) Temperatures (b) pressures (c) pressure difference (b) (d) masses of working fluid.

On the other hand, as the stable regime is reached, the total heat input rate can be calculated by

$$\dot{Q}_{in} = \frac{\omega}{2\pi} \oint \dot{Q}_l^k + \dot{Q}_h^k + \dot{Q}_{r,in}^k dt \quad (20)$$

where $\dot{Q}_l^k = (T_{wl} - T_l^k)/R_l^k$, $\dot{Q}_h^k = (T_{wh} - T_h^k)/R_h^k$, and $\dot{Q}_{r,in}^k$ is the heat input rate calculated from Eq. (10). Furthermore, the thermal efficiency of the engine can be obtained by calculating the ratio of the indicated power to the total heat input rate as:

$$\eta_t = \dot{W}_i / \dot{Q}_{in} \quad (21)$$

The flow chart of computation process is shown in Fig. 5. The computation starts from specifying the initial conditions and stops as the stable cyclic regime is reached. Note that the values of the geometrical and operating parameters listed in Table 1 are taken from the prototype engine shown in Fig. 2. In this study, effects of heating and cooling temperatures, volumes of the chambers, thermal resistances, stroke and bore size on the indicated power output and thermal efficiency are evaluated.

3. Results and discussion

3.1. Base-line case

Fig. 6 shows the stable periodic variation in the thermodynamic properties of the working fluid, including temperatures, pressures, pressure difference, and working fluid masses in the lower- and higher-temperature spaces of the base-line case. In computation, it is observed that the numerical simulation reaches the stable cyclic regime in a number of cycles. Plotted in Fig. 6(a) are the variations in temperatures of the working fluid in the lower- and the higher-temperature spaces. It is noticed that as the wall temperature of the lower-temperature space is fixed at 300 K, the fluid temperature in the space is varied between 360 and 590 K. On the other hand, as the wall temperature of the higher-temperature space is 1000 K, the fluid temperature in the space is varied between 740 and 990 K. The difference between the average temperatures in the two spaces is approximately 400 K. Fig. 6(b) and Fig. 6(c) show the data of pressures in the two spaces and their difference, respectively. It is observed that the pressure difference between the two spaces ($p_l - p_h$) is varied periodically only in a small range from -120 to 90 Pa. Compared to the magnitude of the pressures varied from 0.8 to 4.2 atm, the pressure difference between the two pressures is not

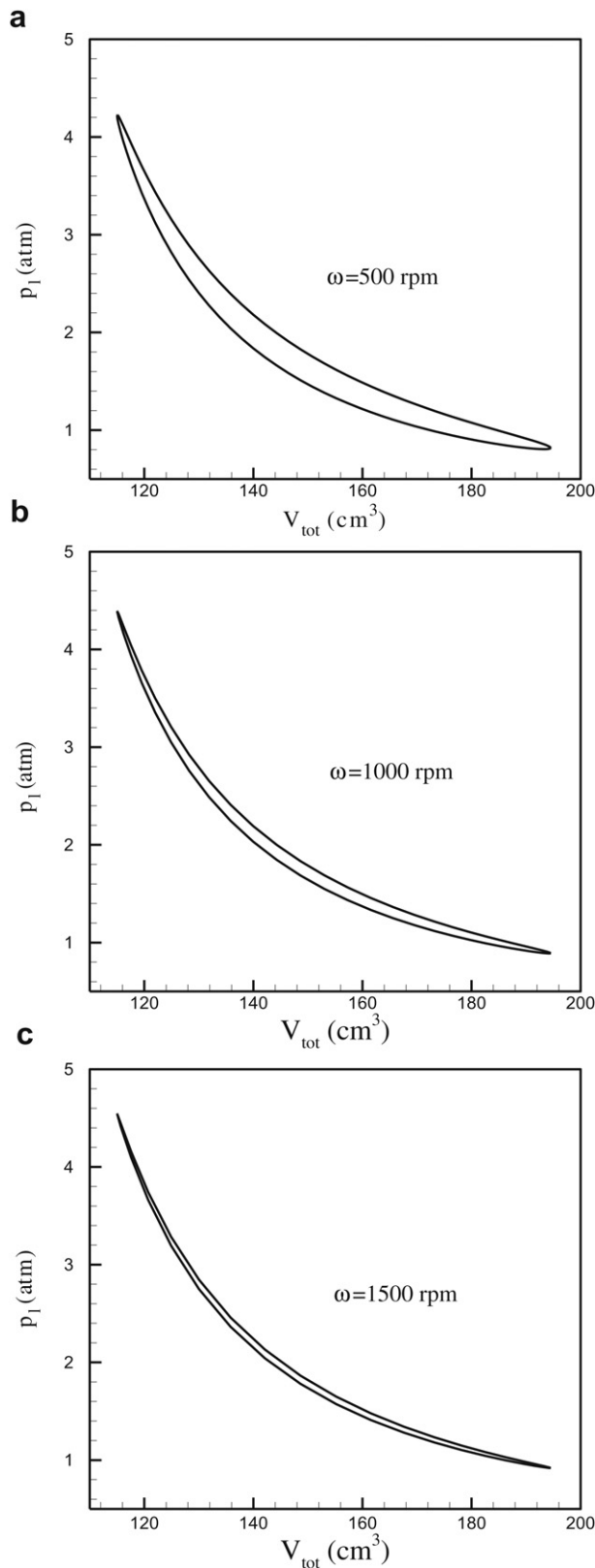


Fig. 7. p – sV diagrams at $\omega = 500$, 1000, and 1500 rpm (a) $\omega = 500$ rpm (b) $\omega = 1000$ rpm (c) $\omega = 1500$ rpm.

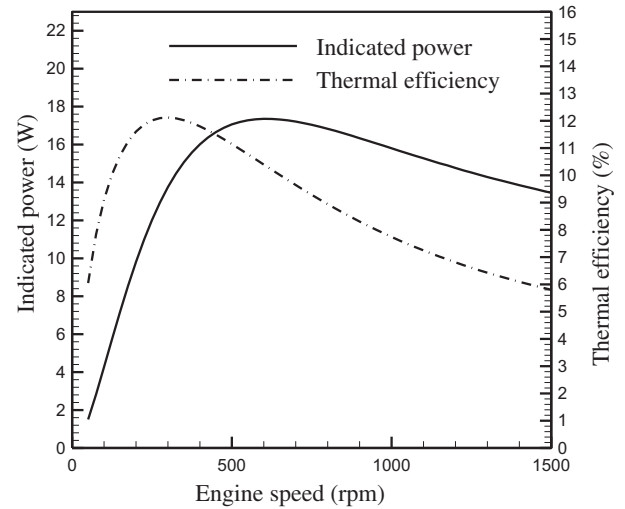


Fig. 8. Dependence of indicated power and thermal efficiency on engine speed.

high relatively. Therefore, the two curves shown in Fig. 6(b) appear to be identical.

Furthermore, the mass flow rate through the regenerative heater is caused by the pressure difference between the two spaces, and hence, the working fluid masses contained in the lower- and the higher-temperature spaces are changed periodically as the pressures are varied. In Fig. 6(d), the variation in the masses contained in the lower- and the higher-temperature spaces is observed. Note that for the base-line case total mass of the working fluid ($m_l + m_h$) is fixed at 7.2×10^{-5} kg even though the masses contained in the individual spaces are varying.

3.2. Parametric study

In the following, a parametric study is performed to evaluate the effects of the geometrical and operating parameters, including the heating and the cooling temperatures, volumes of the chambers, the thermal resistances, and the stroke of the piston, and the bore size on the indicated power output and the thermal efficiency of the engine. In the following parametric study, when a parameter is adjusted to evaluate its effects, other parameters are fixed at the same values given in Table 1.

The stable thermodynamic cycles corresponding to different engine speeds are plotted in the p – V diagrams shown in Fig. 7. In the figure, the engine speed is assigned to be $\omega = 500$, 1000, and 1500 rpm and the stable thermodynamic cycles are plotted in terms of the pressure in lower-temperature space (p_l) and the total volume in the cylinder (V_{tot}). As the piston moves toward the top-dead point, the working fluid is forced to flow into the regenerative heater and the gas chambers, where the temperature and the pressure of the working fluid are increased by heating. The pressure of the working fluid then pushes back the piston and drives the flywheel. As the piston is pushed back toward the bottom-dead point, a part of the working fluid mass flows back to the thermal buffer and the compression chambers for cooling, while another part remains in the regenerative heater and the gas chambers. As more working fluid is cooled in the thermal buffer and the compression chambers, the temperature and the pressure of the working fluid are decreased. Meanwhile, the piston is driven to move toward the top-dead point again to complete a cycle. In the plots, p_l is varied approximately from 0.8 to 4.2 atm. For this particular case at $\omega = 500$ rpm, the indicated power is calculated to be 17.06 W by integrating Eq. (19). It is noticed that the area of the

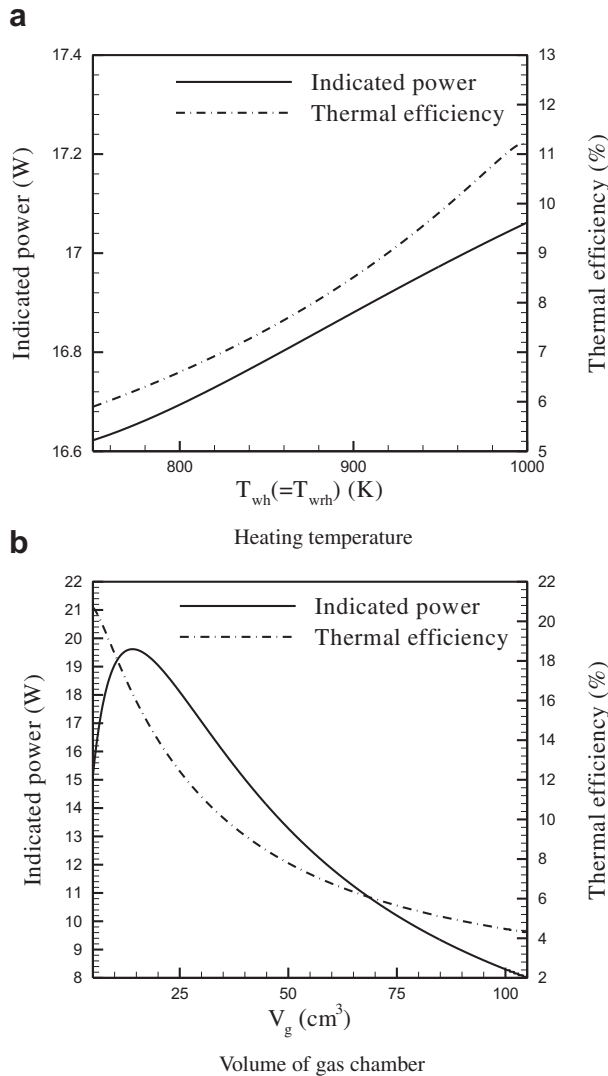


Fig. 9. Effects of higher-temperature space conditions on engine performance. (a) Heating temperature (b) volume of gas chamber.

cycle is decreased as the engine speed is increased. It means that the indicated work per cycle is reduced as the engine speed is elevated. This is reasonable because as the engine speed is elevated, time for heat transfer is shortened and therefore, the amount of heat input per cycle is reduced. The observations on the high-engine-speed cases shown in this figure agree with the data provided by Arques [15] in which the areas of the thermodynamic cycles in the p – V diagrams are very small.

Fig. 8 shows the dependence of the indicated power and the thermal efficiency on the engine speed. Note that though the indicated work per cycle is generally reduced as the engine speed is elevated, the indicated power may still tend to increase with operating speed in the low-speed regime since the indicated power is the product of the indicated work per cycle and the engine speed. It is observed in Fig. 8, when the engine speed is elevated, the indicated power reaches a maximum of 17.35 W at a critical speed of 600 rpm. Over the critical speed, the indicated power falls as the speed is further increased in the high-speed regime. This feature was also found by Cheng and Yang [24] in a study for a α -type Stirling with Ross mechanism. On the other hand, it is found that the thermal efficiency also reaches a maximum value of 12.12% at

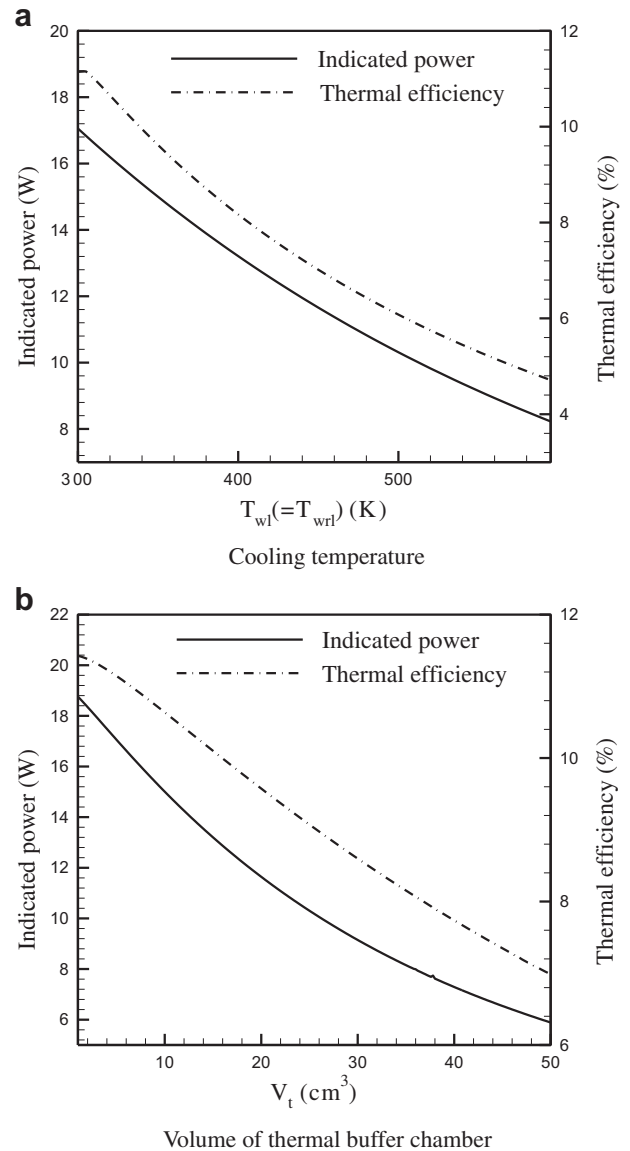


Fig. 10. Effects of lower-temperature space conditions on engine performance. (a) Cooling temperature (b) volume of thermal buffer chamber.

a critical speed of 300 rpm. One of the disadvantages with the thermal lag engine is that its thermal efficiency is relatively low, which can be clearly seen in this figure.

Effects of the higher-temperature space conditions on the performance of the engine are conveyed in Fig. 9. The influence of the heating temperature (T_{wh}) and gas chamber volume (V_g) is illustrated in Fig. 9(a) and (b), respectively. It is observed that both the indicated work and the thermal efficiency increase with the heating temperature monotonically. On the other hand, the thermal efficiency is decreased as the gas chamber volume becomes larger. In Fig. 9(b), it is seen that the thermal efficiency monotonically drops from 21% to 4.4% as the gas chamber volume is increased from 5 to 105 cm³. Furthermore, the gas chamber volume exhibits a subtle influence on the indicated power. It is noticed that there exists an optimal gas chamber volume of 14.2 cm³ with a maximum power of 19.62 W.

Shown in Fig. 10 are the effects of the lower-temperature space conditions on engine performance. In Fig. 10(a), one finds that as the cooling temperature (T_{wl}) is increased, both the indicated work

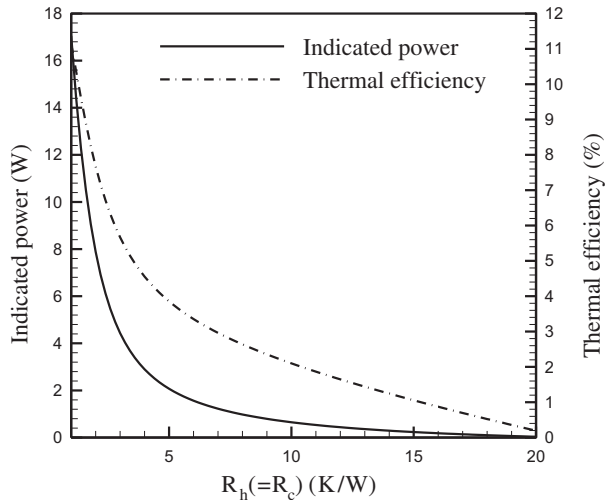


Fig. 11. Effects of thermal resistances on engine performance.

and the thermal efficiency are reduced. According to Fig. 10(a), the performance of the engine will be increased if the cooling temperature is lowered. On the other hand, Fig. 10(b) shows that an increase in the volume of thermal buffer chamber results in decreases in the indicated work and the thermal efficiency. In this figure, it is found that as the volume of thermal buffer chamber is increased from 1 to 50 cm³, the thermal efficiency is reduced from 11.5% to 7.0%, and the indicated work from 18.8 to 6.0 W. This implies that the volume of thermal buffer chamber should be as small as possible in order to have a higher engine performance.

Like all other types of Stirling engines, dead zone is inevitable in the thermal-lag engines. The volumes of the regenerative heater and the thermal buffer chamber are the dead zones that are not swept by the piston. When the piston is moving between the top and the bottom-dead points, some gas will stay within the regenerative heater and the thermal buffer chamber and not able to pass through the regenerative heater for a perfect heating or cooling. This explains why the volume of thermal buffer chamber should be as small as possible in order to have a higher engine performance. Therefore, the imperfect cooling and heating in the lower-temperature and high-temperature spaces, respectively, may be partly attributed to the existence of the dead zones.

Fig. 11 displays the influence of the thermal resistances in the higher-temperature space (R_h) and the compression chamber (R_c) on the engine performance. In practices, the flow situations in the higher-temperature space and in the compression chamber are similar; hence, in the cases shown in this figure, the two thermal resistances are set equal to each other in order to evaluate the dependence of the indicated work and the thermal efficiency on the thermal resistances. Fig. 11 conveys that the indicated work and thermal efficiency can be elevated remarkably by means of reducing the thermal resistances. This is because under the same temperature difference the heat transfer rate is enhanced if the thermal resistance is reduced. It is noticed that the performance of the engine is rather sensitive to the thermal resistances particularly as the thermal resistances are lower than 5.0 K/W.

Finally, effects of geometrical parameters on engine performance are illustrated in Fig. 12. Dependence of engine performance on the stroke of the piston is displayed in Fig. 12(a). It is interesting to find that engine performance increases with the stroke of the piston. As the stroke is increased from 0.03 to 0.08 m, the indicated power is increased from 10.7 to 22.0 W, and the thermal efficiency from 8.5% to 12.0%. It can be attributed to the increase in the swept volume and the compression ratio accompanying the increase in

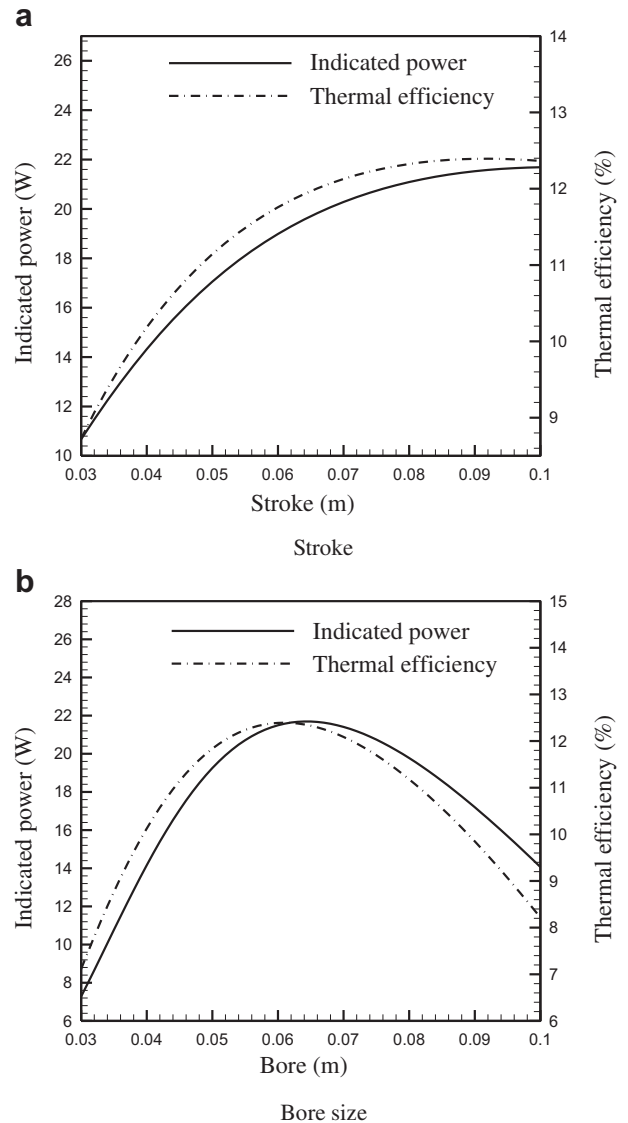


Fig. 12. Effects of geometrical parameters on engine performance. (a) Stroke (b) bore size.

the stroke. However, as the stroke is further increased to a value higher than 0.08 m, only a slight improvement in the engine performance may be obtained.

The swept volume can also rise by increasing the bore size. However, as the bore size is elevated, working fluid mass contained in the cylinder is also increased, and hence, the temperature response of the working fluid may be slowed down. Therefore, there exists a critical bore size for a maximum indicated work or a maximum thermal efficiency. According to Fig. 12(b), the critical bore size for maximum indicated power and maximum thermal efficiency are 0.068 and 0.062 m, respectively.

4. Conclusions

In this study, a numerical model for predicting the thermodynamic cycle of the thermal-lag Stirling engine is built and dependence of the engine performance on the geometrical and operating parameters has been investigated for the base-line case. Results show that though the indicated work per cycle is generally reduced as the engine speed is elevated, the indicated power of the engine

reaches a maximum of 17.35 W at a critical speed of 600 rpm for the particular case.

It is also observed that both the indicated work and the thermal efficiency are elevated by increasing the heating temperature or decreasing the cooling temperature. Meanwhile, it has been seen that the thermal efficiency monotonically drops from 21% to 4.4% as the gas chamber volume is increased from 5 to 105 cm³. In addition, it is found that there exists an optimal gas chamber volume of 14.2 cm³ with a maximum power of 19.62 W. On the other hand, the volume of thermal buffer chamber should be as small as possible in order to have a higher engine performance.

According to the numerical predictions, the indicated work and thermal efficiency can be elevated remarkably by means of reducing the thermal resistances. As to the effects of the geometrical parameters, it is found that engine performance increases with the stroke of the piston; however, as the piston stroke is increased to a value higher than 0.08 m, only a slight improvement in the engine performance may be obtained. There exists a critical bore size for a maximum indicated work or a maximum thermal efficiency. For the present cases, the critical bore size for maximum indicated power and maximum thermal efficiency are 0.068 and 0.062 m, respectively.

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